

**We Claim:**

1. A method of embedding a watermark into an image, said method comprising the step of:

5 maintaining at least one basis pattern; and

adding said basis pattern(s) to said image, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative  
10 constant.

2. A method as claimed in claim 1, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

15 3. A method as claimed in claim 1, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(a.r, \theta + \phi) = c.[g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

20 4. A method as claimed in claim 1, wherein said basis function is a function  $g(r, \theta)$  further defined such that:

$$g(r, \theta) \otimes \{g(a.r, \theta + \phi)\} = [g(r, \theta) \otimes \{c.g(r, \theta)\}]$$

wherein  $N$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

- 5 5. A method as claimed in claim 1, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function.

6. A method as claimed in claim 1, wherein said basis pattern is of the form:

$$g_{pmkn}(r, \theta) = \Re \{ w_n(r, \theta) \cdot r^{i\alpha_m + p} e^{ik\theta} \}$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

7. A method as claimed in claim 6 wherein said window function  $w_n(r, \theta)$  removes or de-emphasises a central region of said basis function having frequencies above a predetermined frequency.

8. A method as claimed in claim 6 wherein said window function  $w_n(r, \theta)$  de-emphasises regions of said basis function corresponding with regions of said image having a low signal variation.

9. A method as claimed in claim 6 wherein said window function  $w_n(r, \theta)$  contains a constant phase factor.

10. A method as claimed in claim 5, wherein at least a first and a second basis patterns are added to said image, with said first and second basis patterns formed from a first and a second basis function respectively, and with at least one parameter  $k$  or  $\alpha_n$  of said first basis function being different to that of said second basis function.

11. A method as claimed in claim 5, wherein a plurality of basis patterns are added with different offsets relative to a center of said image.

12. A method as claimed in claim 11 comprising the further initial step of encoding information into at least one of said parameters, said offset, an amplitude or a relative phase added to said basis pattern.

13. A method of embedding a watermark into an image, said method comprising the step of:

maintaining at least one basis pattern; and

adding said basis pattern(s) to said image, wherein said basis pattern(s) is formed substantially from a real component of a basis function, said basis function is of the form:

$$s_{l,m}(x,y) = w(x,y) |x'|^{i\alpha_l + p_x} |y'|^{i\alpha_m + p_y},$$

wherein  $p_x$ ,  $p_y$ ,  $\alpha_l$  and  $\alpha_m$  are parameters of said basic function,  $w(r,\theta)$  is a window function, and  $x'$  and  $y'$  are predetermined co-ordinates which are rotated relative to the Cartesian co-ordinates  $x$  and  $y$ .

14. A method of embedding a watermark into an image, said method comprising the step of:

maintaining at least one basis pattern; and

5 adding said basis pattern(s) to said image, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

10 15. A method as claimed in claim 14, wherein said basis pattern(s) is a real function - substantially formed from said basis function.

16. A method as claimed in claim 14, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta) = c. [g(r, \theta) \otimes g(r, \theta)]$$

15 wherein  $r$  is a displacement distance,  $\theta$  is an angle,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

17. A method as claimed in claim 14, wherein said basis function is a function  $g(r, \theta)$  further defined such that:

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$$g(r, \theta) \otimes \Re \{g(ar, \theta)\} = [g(r, \theta) \otimes \Re \{c.g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  is an angle,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

18. A method of embedding a watermark into an image, said method comprising the step of:

maintaining at least one basis pattern; and

5 adding said basis pattern(s) to said image, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

10 19. A method as claimed in claim 18, wherein said basis pattern(s) is a real function - substantially formed from said basis function.

20. A method as claimed in claim 19, wherein said basis function is a function  $g(r, \theta)$   
15 defined such that:

$$g(r, \theta) \otimes g(r, \theta + \phi) = c. [g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $\alpha$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

20 21. A method as claimed in claim 19, wherein said basis function is a function  $g(r, \theta)$  further defined such that:

$$g(r, \theta) \otimes \mathbb{N}\{g(r, \theta + \phi)\} = [g(r, \theta) \otimes \mathbb{N}\{c.g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

- 5    22.    A method of detecting a watermark from an image, said watermark having a first basis pattern embedded, said method comprising the steps of:

maintaining a second basis pattern; and

- 10    detecting said first basis pattern in said image using said second basis pattern, said first and second basis patterns being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

- 15    23.    A method as claimed in claim 22, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

24.    A method as claimed in claim 22, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(a.r, \theta + \phi) = c.[g(r, \theta) \otimes g(r, \theta)]$$

- 20    wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

25. A method as claimed in claim 22, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes \Re\{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c.g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

26. A method as claimed in claim 22, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k, p$  and  $\alpha_m$  being parameters of said basis function.

27. A method as claimed in claim 22, wherein said basis pattern is of the form:

$$g_{pmkut}(r, \theta) = \Re\{w_n(r, \theta) \cdot r^{i\alpha_m + p} e^{ik\theta}\}$$

with  $k, p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

28. A method as claimed in claim 22, wherein said detection step comprises the steps of:

correlating said image with said second pattern to form a correlation image; and

locating at least one magnitude peak in said correlation image, said peak corresponding to a centre position where said first basis pattern was embedded into said image.

29. A method as claimed in claim 28, comprising the further final step of decoding information from at least one of said parameters, said peak position(s), an amplitude or relative phase of said peak(s).

5 30. A method as claimed in claim 28, comprising the further initial step of de-emphasising regions of said image having high signal variation.

31. A method of adding registration marks to an image, said method comprising the step of:

10 maintaining at least one basis pattern, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant;

15 adding said basis pattern(s) to said image at at least three predetermined offsets relative to a center of said image.

32. A method as claimed in claim 31, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

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33. A method as claimed in claim 31 wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta + \phi) = c.[g(r, \theta) \otimes g(r, \theta)]$$



wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

34. A method as claimed in claim 31, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes \Re\{g(a.r, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c.g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

35. A method as claimed in claim 31, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $n$ ,  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basic function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis patterns are added to said image.

36. A method of registering a transformed image, wherein a first basis pattern is embedded in said image before transformation at at least three predetermined positions, said method comprising the steps of:

maintaining a second basis pattern;

detecting said first basis pattern in said transformed image using said second basis pattern, said first and second basis patterns being formed substantially from a basis function, wherein said basis function is defined such that said basis function when

correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant;

comparing positions of said first pattern with said predetermined positions;

determining linear transformations for transforming said positions of said first

5 pattern with said predetermined positions; and

transforming said image to invert said linear transformations.

37. A method as claimed in claim 36, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

10 38. A method as claimed in claim 36, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta + \phi) = c[g(r, \theta) \otimes g(r, \theta)]$$

15 wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

39. A method as claimed in claim 36, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes \aleph\{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \aleph\{c.g(r, \theta)\}]$$

20 wherein  $\aleph$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

40. A method as claimed in claim 36, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function.

5 41. A method as claimed in claim 36, wherein said basis pattern is of the form:

$$g_{pmkn}(r, \theta) = \Re \{ w_n(r, \theta) \cdot r^{i\alpha_m + p} e^{ik\theta} \}$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

10 42. A method as claimed in claim 36, wherein said detection step comprises the steps of:

correlating said transformed image with said second pattern to form a correlation image; and

locating at least three magnitude peaks in said correlation image, said peaks

15 determining the positions of said first basis pattern in said transformed image.

43. A method of embedding a watermark into an audio stream, said method comprising the step of:

maintaining at least one basis pattern; and

20 adding said basis pattern(s) to said audio stream, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled version of said basis function is

substantially equal to the auto-correlation of said function within a complex multiplicative constant.

44. A method as claimed in claim 43, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

45. A method as claimed in claim 43, wherein said basis function is a function  $g(t)$  defined such that:

$$g(t) \otimes g(at) = c [g(t) \otimes g(t)]$$

wherein  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

46. A method as claimed in claim 43, wherein said basis function is a function  $g(t)$  defined such that:

$$g(t) \otimes \Re\{g(at)\} = [g(t) \otimes \Re\{c \cdot g(t)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

47. A method as claimed in claim 43, wherein said basis function is of the form:

$$g_{pm}(t) = t^{i\alpha_m + p},$$

with  $p$  and  $\alpha_m$  being parameters of said basis function.

48. A method as claimed in claim 43, wherein said basis pattern is of the form:

$$g_{pmn}(t) = \Re \{ w_n(t) t^{i\alpha_n - p} \}$$

with  $p$  and  $\alpha_n$  being parameters of said basis function, and  $w_n(t)$  is a window function, wherein a real component of said basis pattern is added to said audio stream.

5 49. A method as claimed in claim 48 wherein said window function  $w_n(t)$  removes or de-emphasises a region of said basis function having frequencies above a predetermined frequency.

10 50. A method as claimed in claim 48 wherein said window function  $w_n(t)$  de-emphasises regions of said basis function corresponding with regions of said audio stream having a low signal variation.

15 51. A method of detecting a watermark from an audio stream, said watermark having a first basis pattern embedded, said method comprising the steps of:  
maintaining a second basis pattern; and

detecting said first basis pattern in said audio stream using said second basis pattern, said first and second basis patterns being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

20 52. A method as claimed in claim 51, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

53. A method as claimed in claim 51, wherein said basis function is a function  $g(t)$  defined such that:

$$g(t) \otimes g(at) = c.[g(t) \otimes g(t)]$$

5 wherein  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

54. A method as claimed in claim 51, wherein said basis function is a function  $g(t)$  defined such that:

10 
$$g(t) \otimes \Re\{g(at)\} = [g(t) \otimes \Re\{c.g(t)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

55. A method as claimed in claim 51, wherein said basis function is of the form:

15 
$$g_{pm}(t) = t^{i\alpha_m + p},$$

with  $p$  and  $\alpha_m$  being parameters of said basis function.

56. A method as claimed in claim 51, wherein said basis pattern is of the form:

$$g_{pmn}(t) = \Re\{w_n(t)t^{i\alpha_m + p}\}$$

20 with  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(t)$  is a window function, wherein a real component of said basis pattern is added to said audio stream.

57. A method as claimed in claim 51, wherein said detection step comprises the steps of:

correlating said audio stream with said second pattern to form a correlation signal;

and

5 locating at least one magnitude peak in said correlation signal, said peak corresponding to a position where said first basis pattern was embedded into said audio stream.

58. A method as claimed in claim 56, comprising the further initial step of de-  
10 emphasising regions of said audio stream having high signal variation.

59. An image processing apparatus for embedding a watermark into an image, said apparatus comprising:

means for maintaining at least one basis pattern; and

15 means for adding said basis pattern(s) to said image, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

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60. An apparatus as claimed in claim 59, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

61. An apparatus as claimed in claim 59, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(a.r, \theta + \phi) = c.[g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

62. An apparatus as claimed in claim 59, wherein said basis function  $g(r, \theta)$  further defined such that:

$$g(r, \theta) \otimes \aleph\{g(a.r, \theta + \phi)\} = [g(r, \theta) \otimes \aleph\{c.g(r, \theta)\}]$$

wherein  $\aleph$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

63. An apparatus as claimed in claim 59, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function.

64. An apparatus as claimed in claim 59, wherein said basis pattern is of the form:

$$g_{pmkn}(r, \theta) = \Re \{ w_n(r, \theta) . r^{i\alpha_m + p} e^{ik\theta} \}$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.



65. An apparatus as claimed in claim 64 wherein said window function  $w_n(r, \theta)$  removes or de-emphasises a central region of said basis function having frequencies above a predetermined frequency.

5 66. An apparatus as claimed in claim 64 wherein said window function  $w_n(r, \theta)$  de-emphasises regions of said basis function corresponding with regions of said image having a low signal variation.

67. An apparatus as claimed in claim 64 wherein said window function  $w_n(r, \theta)$   
10 contains a constant phase factor.

68. An apparatus as claimed in claim 63, wherein at least a first and a second basis patterns are added to said image, with said first and second basis patterns formed from a first and a second basis function respectively, and with at least one parameter  $k$  or  $\alpha_n$  of  
15 said first basis function being different to that of said second basis function.

69. An apparatus as claimed in claim 63, wherein a plurality of basis patterns are added with different offsets relative to a center of said image.

20 70. An apparatus as claimed in claim 69 further comprising means for encoding information into at least one of said parameters, said offset, an amplitude or a relative phase added to said basis pattern.

71. An image processing apparatus for embedding a watermark into an image, said apparatus comprising:

means for maintaining at least one basis pattern; and

means for adding said basis pattern(s) to said image, wherein said basis pattern(s)

5 is formed substantially from a real component of a basis function, said basis function is of the form:

$$s_{l,m}(x,y) = w(x,y) |x'|^{i\alpha_l + p_x} |y'|^{i\alpha_m + p_y},$$

wherein  $p_x$ ,  $p_y$ ,  $\alpha_l$  and  $\alpha_m$  are parameters of said basic function,  $w(r,\theta)$  is a window function, and  $x'$  and  $y'$  are predetermined co-ordinates which are rotated relative to the

10 Cartesian co-ordinates  $x$  and  $y$ .

72. An image processing apparatus for embedding a watermark into an image, said apparatus comprising:

means for maintaining at least one basis pattern; and

15 means for adding said basis pattern(s) to said image, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

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73. An apparatus as claimed in claim 72, wherein said basis pattern(s) is a real function - substantially formed from said basis function.

74. An apparatus as claimed in claim 72, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta) = c [g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  is an angle,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

75. An apparatus as claimed in claim 72, wherein said basis function is a function  $g(r, \theta)$  further defined such that:

$$g(r, \theta) \otimes \Re \{g(ar, \theta)\} = [g(r, \theta) \otimes \Re \{c g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  is an angle,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

76. An image processing apparatus for embedding a watermark into an image, said apparatus comprising:

means for maintaining at least one basis pattern; and

means for adding said basis pattern(s) to said image, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

77. An apparatus as claimed in claim 76, wherein said basis pattern(s) is a real function - substantially formed from said basis function.

78. An apparatus as claimed in claim 77, wherein said basis function is a function  
5  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(r, \theta + \phi) = c. [g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

10 79. An apparatus as claimed in claim 77, wherein said basis function is a function  $g(r, \theta)$  further defined such that:

$$g(r, \theta) \otimes \Re \{g(r, \theta + \phi)\} = [g(r, \theta) \otimes \Re \{c.g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles, and  $c$  is a complex number not dependent on said displacement distance  $r$   
15 nor said angle  $\theta$ .

80. An image processing apparatus for detecting a watermark from an image, said watermark having a first basis pattern embedded, said apparatus comprising:

means for maintaining a second basis pattern; and

20 means for detecting said first basis pattern in said image using said second basis pattern, said first and second basis patterns being formed substantially from a basis function, wherein said basis function is defined such that said basis function when

correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

81. An apparatus as claimed in claim 80, wherein said at least one basis pattern is a  
5 real function - substantially formed from said basis function.

82. An apparatus as claimed in claim 80, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta + \phi) = c [g(r, \theta) \otimes g(r, \theta)]$$

10 wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

83. An apparatus as claimed in claim 80, wherein said basis function is a function  $g(r, \theta)$  defined such that:

15 
$$g(r, \theta) \otimes \Re\{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c \cdot g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

20 84. An apparatus as claimed in claim 80, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k, p$  and  $\alpha_m$  being parameters of said basis function.

85. An apparatus as claimed in claim 80, wherein said basis pattern is of the form:

$$g_{pmkn}(r, \theta) = \Re \{ w_n(r, \theta) \cdot r^{i\alpha_n + p} e^{ik\theta} \}$$

with  $k$ ,  $p$  and  $\alpha_n$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

5

86. An apparatus as claimed in claim 80, wherein said means for detection comprises:

means for correlating said image with said second pattern to form a correlation image; and

means for locating at least one magnitude peak in said correlation image, said peak corresponding to a centre position where said first basis pattern was embedded into said image.

10

87. An image processing apparatus for adding registration marks to an image, said apparatus comprising:

means for maintaining at least one basis pattern, said basis pattern(s) being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant;

15

means for adding said basis pattern(s) to said image at at least three predetermined offsets relative to a center of said image.

20

88. An apparatus as claimed in claim 87, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

89. An apparatus as claimed in claim 87 wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(a.r, \theta + \phi) = c.[g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

90. An apparatus as claimed in claim 87, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes \Re\{g(a.r, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c.g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

91. An apparatus as claimed in claim 87, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $n$ ,  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basic function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis patterns are added to said image.

92. An image processing apparatus for registering a transformed image, wherein a first basis pattern is embedded in said image before transformation at at least three predetermined positions, said apparatus comprising:

means for maintaining a second basis pattern;

5 means for detecting said first basis pattern in said transformed image using said second basis pattern, said first and second basis patterns being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant;

10 means for comparing positions of said first pattern with said predetermined positions;

means for determining linear transformations for transforming said positions of said first pattern with said predetermined positions; and

means for transforming said image to invert said linear transformations.

15

93. An apparatus as claimed in claim 92, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

94. An apparatus as claimed in claim 92, wherein said basis function is a function  
20  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta + \phi) = c [g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .



95. An apparatus as claimed in claim 92, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes \Re\{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c.g(r, \theta)\}]$$

5 wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

96. An apparatus as claimed in claim 92, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k, p$  and  $\alpha_m$  being parameters of said basis function.

97. An apparatus as claimed in claim 92, wherein said basis pattern is of the form:

$$g_{pmkn}(r, \theta) = \Re\{w_n(r, \theta) \cdot r^{i\alpha_m + p} e^{ik\theta}\}$$

15 with  $k, p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

98. An apparatus as claimed in claim 92, wherein said means for detection comprises:

means for correlating said transformed image with said second pattern to form a

20 correlation image; and

means for locating at least three magnitude peaks in said correlation image, said peaks determining the positions of said first basis pattern in said transformed image.

99. A data processing apparatus for embedding a watermark into an audio stream, said apparatus comprising:

means for maintaining at least one basis pattern; and

means for adding said basis pattern(s) to said audio stream, said basis pattern(s)

5 being formed substantially from a basis function, wherein said basis function is defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

10 100. An apparatus as claimed in claim 99, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

101. An apparatus as claimed in claim 99, wherein said basis function is a function  $g(t)$  defined such that:

15 
$$g(t) \otimes g(at) = c[g(t) \otimes g(t)]$$

wherein  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

20 102. An apparatus as claimed in claim 99, wherein said basis function is a function  $g(t)$  defined such that:

$$g(t) \otimes \Re\{g(at)\} = [g(t) \otimes \Re\{c.g(t)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

103. An apparatus as claimed in claims 99, wherein said basis function is of the form:

$$g_{pm}(t) = t^{i\alpha_m + p},$$

with  $p$  and  $\alpha_m$  being parameters of said basis function.

5 104. An apparatus as claimed in claim 99, wherein said basis pattern is of the form:

$$g_{pmn}(t) = \Re\{w_n(t)t^{i\alpha_m + p}\}$$

with  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(t)$  is a window function, wherein a real component of said basis pattern is added to said audio stream.

10 105. An apparatus as claimed in claim 104 wherein said window function  $w_n(t)$  removes or de-emphasises a region of said basis function having frequencies above a predetermined frequency.

15 106. An apparatus as claimed in claim 104 wherein said window function  $w_n(t)$  de-emphasises regions of said basis function corresponding with regions of said audio stream having a low signal variation.

107. A data processing apparatus for detecting a watermark from an audio stream, said watermark having a first basis pattern embedded, said apparatus comprising:

20 means for maintaining a second basis pattern; and

means for detecting said first basis pattern in said audio stream using said second basis pattern, said first and second basis patterns being formed substantially from a basis function, wherein said basis function is defined such that said basis function when

correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

108. An apparatus as claimed in claim 107, wherein said at least one basis pattern is a  
5 real function - substantially formed from said basis function.

109. An apparatus as claimed in claim 107, wherein said basis function is a function  $g(t)$  defined such that:

$$g(t) \otimes g(at) = c \cdot [g(t) \otimes g(t)]$$

10 wherein  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

110. An apparatus as claimed in claim 107, wherein said basis function is a function  $g(t)$  defined such that:

$$g(t) \otimes \Re\{g(at)\} = [g(t) \otimes \Re\{c \cdot g(t)\}]$$

15 wherein  $\Re$  defines a real or imaginary component,  $t$  is a displacement time,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement time  $t$ .

111. An apparatus as claimed in claim 107, wherein said basis function is of the form:

$$g_{pm}(t) = t^{i\alpha_m + p},$$

20 with  $p$  and  $\alpha_m$  being parameters of said basis function.

112. An apparatus as claimed in claim 107, wherein said basis pattern is of the form:

$$g_{pmn}(t) = \Re \{ w_n(t) t^{i\alpha_n + p} \}$$

with  $p$  and  $\alpha_n$  being parameters of said basis function, and  $w_n(t)$  is a window function, wherein a real component of said basis pattern is added to said audio stream.

- 5 113. An apparatus as claimed in claim 107, wherein said means for detection comprises:

means for correlating said audio stream with said second pattern to form a correlation signal; and

- 10 means for locating at least one magnitude peak in said correlation signal, said peak corresponding to a position where said first basis pattern was embedded into said audio stream.

114. A program stored in a memory medium for embedding a watermark into an image, said program comprising:

- 15 code for maintaining at least one basis pattern; and

code for adding said basis pattern(s) to said image, wherein said basis pattern(s) is formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex  
20 multiplicative constant.

115. A program as claimed in claim 114, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

116. A program as claimed in claim 114, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta + \phi) = c \cdot [g(r, \theta) \otimes g(r, \theta)]$$

5 wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

117. A program as claimed in claim 114, wherein said basis function is a function  $g(r, \theta)$  defined such that:

10 
$$g(r, \theta) \otimes \Re\{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c \cdot g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

15 118. A program as claimed in claim 114, wherein said basis function is of the form:  
wherein said basis function is a function  $g(r, \theta)$  further defined such that:

$$g(r, \theta) \otimes \Re\{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \Re\{c \cdot g(r, \theta)\}]$$

20 wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

119. A program as claimed in claim 114, wherein a plurality of basis patterns are added with different offsets relative to a center of said image.

120. A program stored in a memory medium for detecting a watermark from an image, said watermark having a first basis pattern embedded, said program comprising:

code for maintaining a second basis pattern; and

5 code for detecting said first basis pattern in said image using said second basis pattern, wherein said first and second basis patterns are formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

10 121. A program as claimed in claim 120, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

122. A program as claimed in claim 120, wherein said basis function is a function  
15  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes g(ar, \theta + \phi) = c [g(r, \theta) \otimes g(r, \theta)]$$

wherein  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $a$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

20 123. A program as claimed in claim 120, wherein said basis function is a function  $g(r, \theta)$  defined such that:

$$g(r, \theta) \otimes \{g(ar, \theta + \phi)\} = [g(r, \theta) \otimes \{c g(r, \theta)\}]$$

wherein  $\Re$  defines a real or imaginary component,  $r$  is a displacement distance,  $\theta$  and  $\phi$  are angles,  $\alpha$  is a positive real number, and  $c$  is a complex number not dependent on said displacement distance  $r$  nor said angle  $\theta$ .

- 5 124. A program as claimed in claim 120, wherein said basis function is of the form:

$$g_{pnk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k, p$  and  $\alpha_m$  being parameters of said basis function.

125. A program as claimed in claim 120, wherein said basis pattern is of the form:

10 
$$g_{pmkn}(r, \theta) = \Re \{ w_n(r, \theta) \cdot r^{i\alpha_m + p} e^{ik\theta} \}$$

with  $k, p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

126. A program as claimed in claim 120, wherein said code for detection comprises:

15 code for correlating said image with said second pattern to form a correlation image; and

code for locating at least one magnitude peak in said correlation image, said peak corresponding to a centre position where said first basis pattern was embedded into said image.

20

127. A program stored in a memory medium for adding registration marks to an image, said program comprising:



code for maintaining at least one basis pattern, wherein said basis pattern(s) is formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant;

code for adding said basis pattern(s) to said image at at least three predetermined offsets relative to a center of said image.

128. A program stored in a memory medium for registering a transformed image, wherein a first basis pattern is embedded in said image before transformation at at least three predetermined positions, said program comprising:

code for maintaining a second basis pattern;

code for detecting said first basis pattern in said transformed image using said second basis pattern, wherein said first and second basis patterns are formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant;

code for comparing positions of said first pattern with said predetermined positions;

code for determining linear transformations for transforming said positions of said first pattern with said predetermined positions; and

code for transforming said image to invert said linear transformations.

129. A program as claimed in claim 128, wherein said code for detection comprises:

code for correlating said transformed image with said second pattern to form a correlation image; and

code for locating at least three magnitude peaks in said correlation image, said peaks determining the positions of said first basis pattern in said transformed image.

5

130. A program stored in a memory medium for embedding a watermark into an audio stream, said program comprising:

code for maintaining at least one basis pattern; and

code for adding said basis pattern(s) to said audio stream, wherein said basis pattern(s) is formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

131. A program stored in a memory medium for detecting a watermark from an audio stream, said watermark having a first basis pattern embedded, said program comprising:

code for maintaining a second basis pattern; and

code for detecting said first basis pattern in said audio stream using said second basis pattern, wherein said first and second basis patterns are formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

132. A program as claimed in claims 131, wherein said code for detection comprises:

code for correlating said audio stream with said second pattern to form a correlation signal; and

code for locating at least one magnitude peak in said correlation signal, said peak corresponding to a position where said first basis pattern was embedded into said audio stream.

133. A watermarked image comprising:

an original image; and

one or more basis patterns added to said original image, wherein said basis pattern(s) is formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled and rotated version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

134. An image as claimed in claim 133, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

135. An image as claimed in claim 133, wherein said basis function is of the form:

$$g_{pmk}(r, \theta) = r^{i\alpha_m + p} e^{ik\theta},$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function..

136. An image as claimed in claim 133, wherein said basis pattern is of the form:

$$g_{pmkn}(r, \theta) = \Re \left\{ w_n(r, \theta) \cdot r^{i\alpha_m + p} e^{ik\theta} \right\}$$

with  $k$ ,  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(r, \theta)$  is a window function, wherein a real component of said basis pattern is added to said image.

137. An audio stream comprising:

5 an original audio stream; and

one or more basis patterns added to said audio stream, wherein said basis pattern(s) is formed substantially from a basis function, said basis function being defined such that said basis function when correlated with a scaled version of said basis function is substantially equal to the auto-correlation of said function within a complex multiplicative constant.

138. An audio stream as claimed in claim 137, wherein said at least one basis pattern is a real function - substantially formed from said basis function.

139. An audio stream as claimed in claim 137, wherein said basis function is of the form:

$$g_{pm}(t) = t^{i\alpha_m + p},$$

with  $p$  and  $\alpha_m$  being parameters of said basis function.

140. An audio stream as claimed in claim 139, wherein said basis pattern is of the form:

$$g_{pmn}(t) = \Re \{ w_n(t) t^{i\alpha_m + p} \}$$

with  $p$  and  $\alpha_m$  being parameters of said basis function, and  $w_n(t)$  is a window function, wherein a real component of said basis pattern is added to said audio stream.